Poverty Rankings of Opportunity Profiles

Vito Peragine* Ernesto Savaglio+,° Stefano Vannucci°

*DTEMQ - University of Bari, +,°DMQTE - Univ. of Pescara, °DEP - University of Siena

December 2008

Motivations

- Poverty is essentially a multidimensional phenomenon (income is just one welfare indicator).
- The necessity to move from an income-based evaluation of social inequities towards a more comprehensive domain
  - Rawls (1971).
  - Sen (1985, 2003).
  - Roemer (1996).
  - Tsui (2002).
- Opportunities as the proper space for distributive judgements.

Existing Literature

- The question of how to compare different opportunity distributions [Kranich (1996), Ok (1997), Savaglio and Vannucci (2007) etc.] has been addressed only in terms of inequality criteria.

Existing Literature

- The question of how to compare different opportunity distributions [Kranich (1996), Ok (1997), Savaglio and Vannucci (2007) etc.] has been addressed only in terms of inequality criteria.
- The question of how to rank different opportunity profiles according to certain poverty rankings has never been addressed before.

Existing Literature

- The question of how to compare different opportunity distributions [Kranich (1996), Ok (1997), Savaglio and Vannucci (2007) etc.] has been addressed only in terms of inequality criteria.
- The question of how to rank different opportunity profiles according to certain poverty rankings has never been addressed before.
- Our approach is:

Existing Literature

- The question of how to compare different opportunity distributions [Kranich (1996), Ok (1997), Savaglio and Vannucci (2007) etc.] has been addressed only in terms of inequality criteria.
- The question of how to rank different opportunity profiles according to certain poverty rankings has never been addressed before.
- Our approach is:
  - Related to the literature on multidimensional poverty measurement (see, Tsui (2002), Chachavarty et al. (1998), Bourguignon and Chakravarty (1999, 2002), but more general of those proposed;

Existing Literature

- The question of how to compare different opportunity distributions [Kranich (1996), Ok (1997), Savaglio and Vannucci (2007) etc.] has been addressed only in terms of inequality criteria.
- The question of how to rank different opportunity profiles according to certain poverty rankings has never been addressed before.
- Our approach is:
    - Related to the literature on multidimensional poverty measurement (see, Tsui (2002), Chachavarty et al. (1998), Bourguignon and Chakravarty (1999, 2002), but more general of those proposed;
    - Axiomatic and we study the logical implications of the properties that a poverty relation on profiles of opportunity sets should satisfy

Existing Literature

- The question of how to compare different opportunity distributions [Kranich (1996), Ok (1997), Savaglio and Vannucci (2007) etc.] has been addressed only in terms of inequality criteria.
- The question of how to rank different opportunity profiles according to certain poverty rankings has never been addressed before.
- Our approach is:
  - Related to the literature on multidimensional poverty measurement (see, Tsui (2002), Chachavarty et al. (1998), Bourguignon and Chakravarty (1999, 2002), but more general of those proposed;
  - Axiomatic and we study the logical implications of the properties that a poverty relation on profiles of opportunity sets should satisfy
- We generalize the most widely used (in the income poverty framework) poverty measures: *head-count ratio*, *poverty-gap*.

Notation and Definitions

Let $N = \{1, .., n\}$ denote the finite set of relevant population units, X an universal nonempty set of opportunities, and $\mathcal{P}[X]$ the set of all *finite* subsets of X. Elements of $\mathcal{P}[X]$ are referred to as *opportunity sets*, and mappings $\mathbf{Y} = (Y_1, ..., Y_n) \in \mathcal{P}[X]^N$ as profiles of opportunity sets, or simply *opportunity profiles*.

- A *poverty threshold* is a set $T \in \mathcal{P}[X]$, which induces a preorder $\succcurlyeq_T^*$on $\mathcal{P}[X]$ by the following rule:

Notation and Definitions

Let $N = \{1, .., n\}$ denote the finite set of relevant population units, X an universal nonempty set of opportunities, and $\mathcal{P}[X]$ the set of all *finite* subsets of X. Elements of $\mathcal{P}[X]$ are referred to as *opportunity sets*, and mappings $\mathbf{Y} = (Y_1, ..., Y_n) \in \mathcal{P}[X]^N$ as profiles of opportunity sets, or simply *opportunity profiles*.

- A *poverty threshold* is a set $T \in \mathcal{P}[X]$, which induces a preorder $\succcurlyeq^*_T$ on $\mathcal{P}[X]$ by the following rule:
  - for any $Y, Z \in \mathcal{P}[X]$, $Y \succcurlyeq^*_T Z$ if and only if $[Y \supseteq Z$ or $Y \supseteq T]$.

Notation and Definitions

Let $N = \{1, .., n\}$ denote the finite set of relevant population units, X an universal nonempty set of opportunities, and $\mathcal{P}[X]$ the set of all *finite* subsets of X. Elements of $\mathcal{P}[X]$ are referred to as *opportunity sets*, and mappings $\mathbf{Y} = (Y_1, ..., Y_n) \in \mathcal{P}[X]^N$ as profiles of opportunity sets, or simply *opportunity profiles*.

- A *poverty threshold* is a set $T \in \mathcal{P}[X]$, which induces a preorder $\succcurlyeq_T^*$ on $\mathcal{P}[X]$ by the following rule:
  - for any $Y, Z \in \mathcal{P}[X]$, $Y \succcurlyeq_T^* Z$ if and only if $[Y \supseteq Z$ or $Y \supseteq T]$.
- A *poverty ranking* of opportunity profiles, under threshold T, is a preorder $\succcurlyeq_T$ on $\mathcal{P}[X]^N$ such that:

Notation and Definitions

Let $N = \{1, .., n\}$ denote the finite set of relevant population units, X an universal nonempty set of opportunities, and $\mathcal{P}[X]$ the set of all *finite* subsets of X. Elements of $\mathcal{P}[X]$ are referred to as *opportunity sets*, and mappings $\mathbf{Y} = (Y_1, ..., Y_n) \in \mathcal{P}[X]^N$ as profiles of opportunity sets, or simply *opportunity profiles*.

- A *poverty threshold* is a set $T \in \mathcal{P}[X]$, which induces a preorder $\succcurlyeq_T^*$on $\mathcal{P}[X]$ by the following rule:
  - for any $Y, Z \in \mathcal{P}[X]$, $Y \succcurlyeq_T^* Z$ if and only if $[Y \supseteq Z$ or $Y \supseteq T]$.
- A *poverty ranking* of opportunity profiles, under threshold T, is a preorder $\succcurlyeq_T$ on $\mathcal{P}[X]^N$ such that:
  - for any $\mathbf{Y}, \mathbf{Z} \in \mathcal{P}[X]^N$, $\mathbf{Y} \succcurlyeq_T \mathbf{Z}$ whenever $Z_i \succcurlyeq_T^* Y_i$ for each $i \in N$.

Notation and Definitions (cont.)

Definition

The *head-count (HC) poverty ranking*, under threshold T, is the preorder $\succsim_T^h$ on $\mathcal{P}[X]^N$ defined as follows:

- for any $\mathbf{Y}, \mathbf{Z} \in \mathcal{P}[X]^N$, $\mathbf{Y} \succsim_T^h \mathbf{Z}$ if and only if $h_T(\mathbf{Y}) \geq h_T(\mathbf{Z})$ where for each $\mathbf{W} \in \mathcal{P}[X]^N$, $h_T(\mathbf{W}) = \#H_T(\mathbf{W})$, and $H_T(\mathbf{W}) = \{i \in N : W_i \not\supseteq T\}$.

Definition

The *opportunity-gap (OG) poverty ranking*, under threshold T, is the preorder $\succsim_T^g$ on $\mathcal{P}[X]^N$ defined as follows:

- for any $\mathbf{Y}, \mathbf{Z} \in \mathcal{P}[X]^N$, $\mathbf{Y} \succsim_T^g \mathbf{Z}$ iff $g_T(\mathbf{Y}) \geq g_T(\mathbf{Z})$, where for each $\mathbf{W} \in \mathcal{P}[X]^N$, $g_T(\mathbf{W}) = \sum_{i \in H_T(\mathbf{W})} \#\{x : x \in T \setminus W_i\}$.

The Axioms

- *Anonymity (A):* For any permutation π of N, and any $\mathbf{Y} \in \mathcal{P}[X]^N$: $\mathbf{Y} \sim_T \pi\mathbf{Y}$ (where $\pi\mathbf{Y} = (Y_{\pi(1)}, ..., Y_{\pi(n)})$).

The Axioms

- *Anonymity (A):* For any permutation π of N, and any $\mathbf{Y} \in \mathcal{P}[X]^N$: $\mathbf{Y} \sim_T \pi\mathbf{Y}$ (where $\pi\mathbf{Y} = (Y_{\pi(1)}, \ldots, Y_{\pi(n)})$).
- *Irrelevance of Inessential Opportunities (I):* For any $\mathbf{Y} \in \mathcal{P}[X]^N$, $i \in N$, and $x \in Y_i \setminus T$: $\mathbf{Y} \sim_T (\mathbf{Y}_{-i}, Y_i \setminus \{x\})$.

The Axioms

- *Anonymity (A):* For any permutation π of N, and any $\mathbf{Y} \in \mathcal{P}[X]^N$: $\mathbf{Y} \sim_T \pi\mathbf{Y}$ (where $\pi\mathbf{Y} = (Y_{\pi(1)}, ..., Y_{\pi(n)})$).
- *Irrelevance of Inessential Opportunities (I):* For any $\mathbf{Y} \in \mathcal{P}[X]^N$, $i \in N$, and $x \in Y_i \setminus T$: $\mathbf{Y} \sim_T (\mathbf{Y}_{-i}, Y_i \setminus \{x\})$.
- *Dominance at Essential Profiles (E):* For any $\mathbf{Y}, \mathbf{Z} \in \mathcal{P}[X]^N$ such that both $\{Y_1, ..., Y_n\} \subseteq \{T, \emptyset\}$ and $\{Z_1, ..., Z_n\} \subseteq \{T, \emptyset\}$, $\mathbf{Y} \succ_T \mathbf{Z}$ if and only if $\# \{i \in N : Y_i = \emptyset\} > \# \{i \in N : Z_i = \emptyset\}$.

The Axioms

- *Anonymity (A):* For any permutation π of N, and any $\mathbf{Y} \in \mathcal{P}[X]^N$: $\mathbf{Y} \sim_T \pi\mathbf{Y}$ (where $\pi\mathbf{Y} = (Y_{\pi(1)}, ..., Y_{\pi(n)})$).
- *Irrelevance of Inessential Opportunities (I):* For any $\mathbf{Y} \in \mathcal{P}[X]^N$, $i \in N$, and $x \in Y_i \setminus T$: $\mathbf{Y} \sim_T (\mathbf{Y}_{-i}, Y_i \setminus \{x\})$.
- *Dominance at Essential Profiles (E):* For any $\mathbf{Y}, \mathbf{Z} \in \mathcal{P}[X]^N$ such that both $\{Y_1, ..., Y_n\} \subseteq \{T, \emptyset\}$ and $\{Z_1, ..., Z_n\} \subseteq \{T, \emptyset\}$, $\mathbf{Y} \succ_T \mathbf{Z}$ if and only if $\#\{i \in N : Y_i = \emptyset\} > \#\{i \in N : Z_i = \emptyset\}$.
- *Irrelevance of Poor' Opportunity Deletions (D):* For any $\mathbf{Y} \in \mathcal{P}[X]^N$, $i \in H_T(\mathbf{Y})$, and $x \in Y_i$: $\mathbf{Y} \sim_T (\mathbf{Y}_{-i}, Y_i \setminus \{x\})$.

The Axioms

- *Anonymity (A):* For any permutation π of N, and any $\mathbf{Y} \in \mathcal{P}[X]^N$: $\mathbf{Y} \sim_T \pi\mathbf{Y}$ (where $\pi\mathbf{Y} = (Y_{\pi(1)}, ..., Y_{\pi(n)})$).
- *Irrelevance of Inessential Opportunities (I):* For any $\mathbf{Y} \in \mathcal{P}[X]^N$, $i \in N$, and $x \in Y_i \setminus T$: $\mathbf{Y} \sim_T (\mathbf{Y}_{-i}, Y_i \setminus \{x\})$.
- *Dominance at Essential Profiles (E):* For any $\mathbf{Y}, \mathbf{Z} \in \mathcal{P}[X]^N$ such that both $\{Y_1, ..., Y_n\} \subseteq \{T, \emptyset\}$ and $\{Z_1, ..., Z_n\} \subseteq \{T, \emptyset\}$, $\mathbf{Y} \succ_T \mathbf{Z}$ if and only if $\#\{i \in N : Y_i = \emptyset\} > \#\{i \in N : Z_i = \emptyset\}$.
- *Irrelevance of Poor' Opportunity Deletions (D):* For any $\mathbf{Y} \in \mathcal{P}[X]^N$, $i \in H_T(\mathbf{Y})$, and $x \in Y_i$: $\mathbf{Y} \sim_T (\mathbf{Y}_{-i}, Y_i \setminus \{x\})$.
- *Strict Monotonicity with respect to Essential Deletions (M):* For any $\mathbf{Y} \in \mathcal{P}[X]^N$, $i \in N$, and $x \in Y_i \cap T$: $(\mathbf{Y}_{-i}, Y_i \setminus \{x\}) \succ_T \mathbf{Y}$.

The Axioms

- *Anonymity (A):* For any permutation π of N, and any $\mathbf{Y} \in \mathcal{P}[X]^N$: $\mathbf{Y} \sim_T \pi\mathbf{Y}$ (where $\pi\mathbf{Y} = (Y_{\pi(1)}, ..., Y_{\pi(n)})$).
- *Irrelevance of Inessential Opportunities (I):* For any $\mathbf{Y} \in \mathcal{P}[X]^N$, $i \in N$, and $x \in Y_i \setminus T$: $\mathbf{Y} \sim_T (\mathbf{Y}_{-i}, Y_i \setminus \{x\})$.
- *Dominance at Essential Profiles (E):* For any $\mathbf{Y}, \mathbf{Z} \in \mathcal{P}[X]^N$ such that both $\{Y_1, ..., Y_n\} \subseteq \{T, \emptyset\}$ and $\{Z_1, ..., Z_n\} \subseteq \{T, \emptyset\}$, $\mathbf{Y} \succ_T \mathbf{Z}$ if and only if $\# \{i \in N : Y_i = \emptyset\} > \# \{i \in N : Z_i = \emptyset\}$.
- *Irrelevance of Poor' Opportunity Deletions (D):* For any $\mathbf{Y} \in \mathcal{P}[X]^N$, $i \in H_T(\mathbf{Y})$, and $x \in Y_i$: $\mathbf{Y} \sim_T (\mathbf{Y}_{-i}, Y_i \setminus \{x\})$.
- *Strict Monotonicity with respect to Essential Deletions (M):* For any $\mathbf{Y} \in \mathcal{P}[X]^N$, $i \in N$, and $x \in Y_i \cap T$: $(\mathbf{Y}_{-i}, Y_i \setminus \{x\}) \succ_T \mathbf{Y}$.
- *Independence of Balanced Essential Deletions (B):* For any $\mathbf{Y}, \mathbf{Z} \in \mathcal{P}[X]^N$, $i \in N$, $y \in Y_i \cap T$ and $z \in Z_i \cap T$:

$$\mathbf{Y} \succcurlyeq_T \mathbf{Z} \text{ if and only if } (\mathbf{Y}_{-i}, Y_i \setminus \{y\}) \succcurlyeq_T (\mathbf{Z}_{-i}, Z_i \setminus \{z\}).$$

Results

- PROPOSITION 1: The binary relational system $(\mathcal{P}[X]^N, \succcurlyeq_T)$, where $\succcurlyeq_T$ is a poverty ranking of $\mathcal{P}[X]^N$ under threshold $T \subseteq X$, satisfies I, D, E and A if and only if $\succcurlyeq_T = \succcurlyeq_T^h$.

- PROPOSITION 1: The binary relational system $(\mathcal{P}[X]^N, \succcurlyeq_T)$, where $\succcurlyeq_T$ is a poverty ranking of $\mathcal{P}[X]^N$ under threshold $T \subseteq X$, satisfies I, D, E and A if and only if $\succcurlyeq_T = \succcurlyeq_T^h$.
- PROPOSITION 2: The binary relational system $(\mathcal{P}[X]^N, \succcurlyeq_T)$, with $\succcurlyeq_T$ a poverty ranking on $\mathcal{P}[X]^N$ under threshold $T \subseteq X$, satisfies I, B, A,M if and only if $\succcurlyeq_T = \succcurlyeq_T^g$.

Composite Rankings

Definition

A (HG)- *lexicographic poverty ranking* of opportunity profiles, under threshold T, is a binary relational system $\left(\mathcal{P}[X]^N, \succsim_T^{hg}\right)$ where $\succsim_T^{hg}$ is a preorder defined as follow: for any $\mathbf{Y}, \mathbf{Z} \in \mathcal{P}[X]^N$,

$$\mathbf{Y} \succsim_T^{hg} \mathbf{Z} \text{ if and only if either } \mathbf{Y} \succ_T^{h} \mathbf{Z} \text{ or } (\mathbf{Y} \sim_T^{h} \mathbf{Z} \text{ and } g_T(\mathbf{Y}) \geq g_T(\mathbf{Z})).$$

Definition

A (GH)- *lexicographic poverty ranking* of opportunity profiles, under threshold T, is a binary relational system $\left(\mathcal{P}[X]^N, \succsim_T^{gh}\right)$ where $\succsim_T^{gh}$ is a preorder defined as follow: for any $\mathbf{Y}, \mathbf{Z} \in \mathcal{P}[X]^N$,

$$\mathbf{Y} \succsim_T^{gh} \mathbf{Z} \text{ if and only if either } \mathbf{Y} \succ_T^{g} \mathbf{Z} \text{ or } (\mathbf{Y} \sim_T^{g} \mathbf{Z} \text{ and } h_T(\mathbf{Y}) \geq h_T(\mathbf{Z})).$$

Composite Rankings (cont.)

Definition

A (HG)-*weighted poverty ranking* of opportunity profiles, under threshold T, is a binary relational system $\left(\mathcal{P}[X]^N, \succcurlyeq_T^w\right)$ where $\succcurlyeq_T^w$ is a preorder defined as follow: there exist w_1, $w_2 \in \mathbb{R}_{++}$ such that, for any $\mathbf{Y}, \mathbf{Z} \in \mathcal{P}[X]^N$, $\mathbf{Y} \succcurlyeq_T^w \mathbf{Z}$ if and only if

$$w_1 h_T(\mathbf{Y}) + w_2 g_T(\mathbf{Y}) \geq w_1 h_T(\mathbf{Z}) + w_2 g_T(\mathbf{Z}).$$

More Axioms

- *Qualified Independence of Balanced Essential Deletions* $(Q-B)$: For any $\mathbf{Y}, \mathbf{Z} \in \mathcal{P}[X]^N$, for any $x, y, z \in X$ and for any $i \in N$, such that $Y_i \subset T$, $Z_i \subset T$, $y \in Y_i \cap T$ and $z \in Z_i \cap T$:$\mathbf{Y} \succcurlyeq_T \mathbf{Z}$ if and only if $(\mathbf{Y}_{-i}, Y_i \setminus \{y\}) \succcurlyeq_T (\mathbf{Z}_{-i}, Z_i \setminus \{z\})$.

More Axioms

- *Qualified Independence of Balanced Essential Deletions* $(Q-B)$: For any $\mathbf{Y},\mathbf{Z}\in\mathcal{P}[X]^N$, for any $x,y,z\in X$ and for any $i\in N$, such that $Y_i\subset T$, $Z_i\subset T$, $y\in Y_i\cap T$ and $z\in Z_i\cap T$:$\mathbf{Y}\succcurlyeq_T\mathbf{Z}$ if and only if $(\mathbf{Y}_{-i},Y_i\setminus\{y\})\succcurlyeq_T(\mathbf{Z}_{-i},Z_i\setminus\{z\})$.
- *Conditional Dominance* (CD) : Let $\succcurlyeq_T$ be a poverty ranking with threshold T. Suppose there exist a positive integer k and $f_1,...,f_k\in\mathbb{R}^{\mathcal{P}[X]^N}$, such that for all $\mathbf{Y},\mathbf{Z}\in\mathcal{P}[X]^N$, $f_i(\mathbf{Y})=f_i(\mathbf{Z})$, $i=1,...,k$ entails $\mathbf{Y}\sim_T\mathbf{Z}$. Then, for all $\mathbf{Y},\mathbf{Z}\in\mathcal{P}[X]^N$, $(f_1(\mathbf{Y}),...,f_k(\mathbf{Y}))\neq(f_1(\mathbf{Z}),...,f_k(\mathbf{Z}))$ and $f_i(\mathbf{Y})\geq f_i(\mathbf{Z})$, $i=1,...,k$ entails $\mathbf{Y}\succ_T\mathbf{Z}$.

More Axioms (cont.)

- *Non-Compensation* (NC) : Let $\succcurlyeq_T$ be a poverty ranking with threshold T. Suppose there exist a positive integer k and $f_1, ..., f_k \in \mathbb{R}^{\mathcal{P}[X]^N}$, such that:
  (i) for all $\mathbf{Y}, \mathbf{Z} \in \mathcal{P}[X]^N$: if $f_i(\mathbf{Y}) = f_i(\mathbf{Z})$, $i = 1, ..., k$, then $\mathbf{Y} \sim_T \mathbf{Z}$,
  (ii) there exist $\mathbf{Y}, \mathbf{Z} \in \mathcal{P}[X]^N$ and $i^* \in \{1, ..., k\}$, such that $f_{i^*}(\mathbf{Y}) > f_{i^*}(\mathbf{Z})$ and $f_j(\mathbf{Z}) > f_j(\mathbf{y})$for any $j \in \{1, ..., k\}$, $j \neq i^*$, and $\mathbf{Y} \succ_T \mathbf{Z}$.
  Then for all $\mathbf{U}, \mathbf{V} \in \mathcal{P}[X]^N$: $\mathbf{U} \succ_T \mathbf{V}$ whenever $f_{i^*}(\mathbf{U}) > f_{i^*}(\mathbf{V})$.

More Axioms (cont.)

- *Head-Count Priority* (HP) : Let $\succcurlyeq_T$ be a poverty ranking with threshold T, such that $\#T \geq 3$. For any $\mathbf{Y}, \mathbf{Z} \in \mathcal{P}[X]^N$, if **[**there exist $i, j \in N$ and $x, y, z \in T$, with $x \neq y \neq z \neq x$, such that for any $l \neq i, j$, $Y_l = Z_l$, $Y_i = T \setminus \{x\}$, $Y_j = T \setminus \{y\}$, $Z_i = T$, and $Z_j = T \setminus \{x, y, z\}$**]**,then $\mathbf{Y} \succ_T \mathbf{Z}$.

More Axioms (cont.)

- *Head-Count Priority* (HP) : Let $\succsim_T$ be a poverty ranking with threshold T, such that $\#T \geq 3$. For any $\mathbf{Y}, \mathbf{Z} \in \mathcal{P}[X]^N$, if **[**there exist $i, j \in N$ and $x, y, z \in T$, with $x \neq y \neq z \neq x$, such that for any $l \neq i, j$, $Y_l = Z_l$, $Y_i = T \setminus \{x\}$, $Y_j = T \setminus \{y\}$, $Z_i = T$, and $Z_j = T \setminus \{x, y, z\}$**]**,then $\mathbf{Y} \succ_T \mathbf{Z}$.
- *Gap-Priority* (GP) : Let $\succsim_T$ be a poverty ranking with threshold T, such that $\#T \geq 3$. For any $\mathbf{Y}, \mathbf{Z} \in \mathcal{P}[X]^N$, if [there exist $i, j \in N$ and $x, y, z \in T$, with $x \neq y \neq z \neq x$, such that for any $l \neq i, j$, $Y_l = Z_l$, $Y_i = T \setminus \{x\}$, $Y_j = T \setminus \{y\}$, $Z_i = T$, and $Z_j = T \setminus \{x, y, z\}$],then $\mathbf{Z} \succ_T \mathbf{Y}$.

More Axioms (cont.)

- *Cardinal Unit-Comparability* (CUC) : Let $\succcurlyeq_T$ be a poverty ranking with threshold T. Suppose there exist a positive integer k and $f_1, ..., f_k \in \mathbb{R}^{\mathcal{P}[X]^N}$, such that for all $\mathbf{Y}, \mathbf{Z} \in \mathcal{P}[X]^N$: if $f_i(\mathbf{Y}) = f_i(\mathbf{Z})$, $i = 1, ..., k$ entails $\mathbf{Y} \sim_T \mathbf{Z}$. Posit

$$\Phi = \left\{ \begin{array}{c} \varphi = (\varphi_1, ..., \varphi_k) : \varphi_i \in \mathbb{R}^{\mathbb{R}}, i : 1, ..., k \text{ such that there exist} \\ \alpha > 0, \beta_i \in \mathbb{R} \text{ with } \varphi_i(x) = \alpha x + \beta_i \text{ for any } x \in \mathbb{R} \end{array} \right\}.$$

Then, for all $\mathbf{Y}, \mathbf{Z}, \mathbf{V}, \mathbf{U} \in \mathcal{P}[X]^N$, $\mathbf{Y} \succcurlyeq_T \mathbf{Z}$ and $(f_1(\mathbf{U}), ..., f_k(\mathbf{U})) = ((\varphi_1 \circ f_1)(\mathbf{Y}), ..., (\varphi_k \circ f_k)(\mathbf{Y}))$ and $(f_1(\mathbf{V}), ..., f_k(\mathbf{V})) = ((\varphi_1 \circ f_1)(\mathbf{Z}), ..., (\varphi_k \circ f_k)(\mathbf{Z}))$ with $\varphi = (\varphi_1, ..., \varphi_k) \in \Phi$ entail $\mathbf{U} \succcurlyeq_T \mathbf{V}$.

More results

Lemma

Let $\succcurlyeq_T$ be a poverty ranking on $\mathcal{P}[X]^N$ and a total preorder which satisfies A, I, D and $Q - B$. Then, for any $\mathbf{Y}, \mathbf{Z} \in \mathcal{P}[X]^N$, $(h_T(\mathbf{Y}), g_T(\mathbf{Y})) = (h_T(\mathbf{Z}), g_T(\mathbf{Z}))$ entails $\mathbf{Y} \sim_T \mathbf{Z}$.

More results (cont.)

- PROPOSITION 3: Let $\succsim_T$ be a poverty ranking of $\mathcal{P}[X]^N$ under threshold $T \subseteq X$, such that $\#T \geq 3$, and a total preorder. Then, $\succsim_T = \succsim_T^{hg}$ if and only if $\succsim_T$ satisfies *A*, *I*, *D*, *Q-B*, *CD*, *NC*, and *HP*.

More results (cont.)

- PROPOSITION 3: Let $\succsim_T$ be a poverty ranking of $\mathcal{P}[X]^N$ under threshold $T \subseteq X$, such that $\#T \geq 3$, and a total preorder. Then, $\succsim_T = \succsim_T^{hg}$ if and only if $\succsim_T$ satisfies *A*, *I*, *D*, *Q-B*, *CD*, *NC*, and *HP*.
- PROPOSITION 4: Let $\succsim_T$ be a poverty ranking of $\mathcal{P}[X]^N$ under threshold $T \subseteq X$ such that $\#T \geq 3$, and a total preorder. Then, $\succsim_T = \succsim_T^{gh}$ if and only if $\succsim_T$ satisfies *A*, *I*, *D*, *Q-B*, *CD*, *NC*, and *GP*.

More results (cont.)

- PROPOSITION 3: Let $\succcurlyeq_T$ be a poverty ranking of $\mathcal{P}[X]^N$ under threshold $T \subseteq X$, such that $\#T \geq 3$, and a total preorder. Then, $\succcurlyeq_T = \succcurlyeq_T^{hg}$ if and only if $\succcurlyeq_T$ satisfies *A*, *I*, *D*, *Q-B*, *CD*, *NC*, and *HP*.
- PROPOSITION 4: Let $\succcurlyeq_T$ be a poverty ranking of $\mathcal{P}[X]^N$ under threshold $T \subseteq X$ such that $\#T \geq 3$, and a total preorder. Then,$\succcurlyeq_T = \succcurlyeq_T^{gh}$if and only if $\succcurlyeq_T$ satisfies *A*, *I*, *D*, *Q-B*, *CD*, *NC*, and *GP*.
- PROPOSITION 5: Let $\succcurlyeq_T$ be a poverty ranking of $\mathcal{P}[X]^N$ under threshold $T \subseteq X$ and a total preorder, and suppose $n > \#T \geq 2$. Then, $\succcurlyeq_T = \succcurlyeq_T^{w}$ if and only if $\succcurlyeq_T$ satisfies *A*, *I*, *D*, *Q-B*, *CD* and *CUC*.

Final Remarks

- A possible extension of our analysis would be to compare the opportunties available to societies with *different numbers of individuals*. This would make it possible to rank opportunity profiles for different countries, different demographic groups, and for different time periods.
- Further, to extend the standard classes of poverty indices to our setting, where the comparison of opportunity sets is induced by a (so-called) *desiderability relation*.