Multidimensional Poverty:

Restricted and Unrestricted Hierarchy among Poverty Dimensions[§]

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Abstract

The increasing interest in multidimensional poverty and well-being analysis added complexity to the way these phenomena are conceptualized and measured. A further source of arbitrariness typically derives from the choice of the weights to be attached to poverty dimensions. In the literature there has not been thus far a specific attempt to conceptualise the nature of the desired hierarchy among the selected poverty dimensions. That is, the possible meanings of the statement "dimension *h* is more important than dimension *k*" have not critically been searched for. The aim of this paper is to move a first step into that direction. We envisage two simple and highly alternative ways in which such a statement can be understood, which we label *restricted* and *unrestricted hierarchy*. The analytical conditions allowing to incorporate them into a poverty index are derived and their implications in terms of the understanding of poverty are discussed. An empirical application shows how the choice of the hierarchical scheme for poverty dimensions can lead to opposite conclusions on the trend of poverty.

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1 Introduction

The assertion that poverty should be conceived and measured in a multidimensional setting has received a widespread agreement in the literature and captured the attention of researchers, institutions and policymakers. At a conceptual level, Amartya Sen's seminal work in the '80s represents a turning point regarding the desirability to extend the attention beyond income and economic resources and to conceive poverty as a condition of functioning failures or lack of opportunities in a multidimensional setting. However, at a measurement level, it is principally in the last decade that scholars addressed their efforts to this field of investigation and significant contributions have been offered. These include i) multidimensional poverty indices (Tsui, 2002; Bourguignon and Chakravarty, 1999, 2003; Alkire and Foster, 2008, among others)[1], ii) non-aggregative strategies such as multidimensional poverty orderings and stochastic dominance (see, in particular, Bourguignon and Chakravarty, 2002; Atkinson, 2003; Duclos, Sahn and Younger, 2006) and iii) multidimensional poverty analysis based on the use of multivariate statistical techniques (Krishnakumar, 2007; Asselin and Tuan Anh, 2008, Krishnakumar and Ballon 2008). Despite the growing attention paid to multidimensionality in poverty measurement, many issues are still open to discussion and thus far most empirical work on poverty is still largely based on the unidimensional income (or consumption) space.

As in the unidimensional case, multidimensional poverty measurement involves a series of value judgements such as, *inter alia*, the choice of the poverty index and the level at which the poverty thresholds should be set up. However, when multidimensionality comes into the picture, additional arbitrariness arises regarding the selection of the dimensions deemed relevant for poverty evaluation, the method of aggregation across dimensions as well as the relative importance to be assigned to each of them. It is particularly on this last issue that this paper will focus.

The typical identification and aggregation steps that characterise any poverty analysis become much more articulated and complex when many dimensions or attributes are involved. Multidimensional identification requires to define first who is poor in each domain and second in how many dimensions an individual should be deprived for being classified as poor – what Alkire and Foster (2008) define as dual cut-off method of identification. While the first stage of the identification procedure simply requires to replicate the unidimensional case choosing idiosyncratic poverty thresholds for each dimension, the second stage is specific to multidimensional analysis. The possible options considered in literature range from union to intersection approaches, where the

[1] As Brandolini (2007) outlines, the use of multidimensional indexes should be distinguished from what he classifies as *fully aggregative strategy*, based on the construction of a composite index of well-being at individual level in a way that poverty evaluation can be brought back to the unidimensional space (Maasoumi and Nickelsburg, 1988, Deutsch and Silber, 2005, Ramos 2008). This approach entails the use of a single poverty line, while for multidimensional indexes an idiosyncratic threshold for each dimension has to be identified.

criterion for the classification of an individual as poor is a situation of deprivation in, respectively, at least one dimension or in all dimensions. An alternative option frequently used in empirical studies is to adopt a so-called 'counting approach', where the poverty level of an individual is represented by the number of dimensions in which she is poor (Townsend, 1979; Mack and Lindsay, 1985. Callan et al, 1999).

Multidimensional aggregation requires specifying a functional form for a multidimensional poverty index and choosing which weighting scheme will determine the relative importance of the selected poverty dimensions. In empirical studies different rationales have been adopted to choose a weighting scheme, ranging from weights chosen by the analyst to statistically-derived weights (both frequency-based and derived through multivariate techniques) or survey-based weights. The choice of a weighting scheme is undoubtedly a task of major importance for a cogent evaluation of multidimensional poverty since different weighting systems may well lead to opposite results and have obvious consequences in terms of policy implications (Brandolini, 2007). Besides, it is an *inescapable* step, because, it may definitely be argued, the act of 'not giving weights' – equivalent indeed to the assignation of identical weights to each dimension – is itself a subjective decision motivated by the value judgement that those dimensions are equally valuable. We therefore follow the invitation of both Sen (1973) and Atkinson (1987) not to turn the necessary awareness of the arbitrariness involved in the exercise into a nihilistic attitude leading to disregard what we *can* say.

In the literature there has not been thus far a specific attempt to conceptualise the nature of the desired hierarchy among the selected poverty dimensions. In other words, the possible meanings of the statement "dimension *h* is more important than dimension *k*" have not critically been searched for. The paper moves a first step in that direction by conceptualising two alternative, simple and highly intuitive ways in which such statement can be understood: *restricted* and *unrestricted hierarchy*.

The paper develops as follows. In Section 2 we present the notation used throughout the paper and describe the value judgements informing customary poverty axioms. In Section 3 we firstly introduce our newly conceptualised two types of hierarchy and illustrate the intuitions behind them. Secondly, we derive the analytical implications for their accommodation by a poverty index, namely an impossibility theorem for what concerns the simultaneous accommodation of unrestricted hierarchy and continuity at the poverty line and a characterisation theorem for the functional form allowing the accommodation of unrestricted hierarchy. Interesting implications in terms of the understanding of poverty and welfare are explored. In Section 4 we show how the above poverty criteria can be easily incorporated in empirical studies through individual deprivation functions based upon poverty dimensions' rank order. Further, we carry out an application of our methodology using microdata for Maldives in 1997 and 2004, providing empirical evidence that the verdict of multidimensional poverty comparisons can be completely reversed under the two types of hierarchy. Section 5 concludes.

2 Multidimensional poverty indices: basic notation and properties

We refer to $\mathbb{N}$, $\mathbb{R}_+$ and $\mathbb{R}_{++}$ as to the sets of strictly positive integers, nonnegative and strictly positive real numbers, respectively. In a society of size $n \in \mathbb{N}$, the typical i_{th} individual possesses $m \in \mathbb{N}$ attributes identified by a vector $x_i = (x_{i,1}, x_{i,2}, ..., x_{i,m})$ whose values are thrown from $\mathbb{R}^m_+$, the *m*-dimensional nonnegative Euclidean space. Each vector of individual attributes can be thought of as one of the *n* rows of a $n \times m$ matrix $X \in M^n$, where M^n denotes the set of all conceivable $n \times m$ matrices whose entries are nonnegative real numbers. Let $M = \bigcup_{n=1}^{\infty} M^n$, and let a vector $z = (z_1, z_2, ..., z_m) \in Z$ exhibit for each dimension the threshold below which an individual is considered poor in that dimension,[2] where Z is a subset of the *m*-dimensional nonnegative Euclidean space with the origin deleted. The multidimensional poverty level in society is obtained by means of multidimensional indices π which are mathematical functions mapping arguments such as individual attributes belonging to matrix *X* and poverty thresholds belonging to the set *Z* to the nonnegative orthant of the real line – i.e. $\pi : M \times Z \to \mathbb{R}_+$. As the desirability of any social indicator rests on the extent to which the value judgements motivating the properties it possesses are shared, let us look at the properties met by the multidimensional poverty index:

$$\pi = \frac{1}{n} \sum_{j=1}^{m} \sum_{i \in Q_j} p_{i,j} , \qquad (1)$$

where Q_j denotes the set of individuals that are poor in dimension j[3] and the poverty level of individual *i* in dimension *j* is given by the non-increasing, positive- and real-valued deprivation function $p_{i,j} = p_j(x_{i,j}, z_j)$. Building upon the results of Foster and Shorrocks (1991) in the unidimensional space, the additive form of π allows the accommodation of the *Subgroup Consistency* axioms: an increase (decrease) in the poverty level of a subset of the population

[2] In this way for each dimension we follow the *weak* definition of the poor provided by Donaldson and Weymark (1986) – the *strong* definition consisting in deeming poor also those individuals which are at the poverty line.

[3] For an exhaustive coverage of the issue of the identification-counting of the poor in the multidimensional space see Bourguignon and Chakravarty (2003) and Alkire and Foster (2008).

induces an increase (decrease) in the aggregate poverty figure. π satisfies also the *Anonymity* axiom – permutations of the entire array of attributes across individuals leave the aggregated poverty level unchanged – and the *Population Invariance* axiom – if two identical societies are merged then the poverty level in the resulting society equals the poverty level in each of the original societies. Furthermore, when π is insensitive to the distribution of attributes above the respective poverty thresholds the index is said to satisfy the *Focus* axiom.

Among the multidimensional indices possessing the properties mentioned above, let us consider the family $\tilde{\pi}$ where the individual deprivation function is $p_{i,j} = p_j\left(t_{i,j}\right)$, $t_{i,j} = \frac{x_{i,j}}{z_j}$. That functional form ensures the accommodation of the *Scale Invariance* axiom, according to which the multiplication of both the j_{th} column of attributes in the matrix X and the relevant poverty threshold by a positive scalar leaves the index unaffected. It should be noted that $p_{i,j}$ in $\tilde{\pi}$ is a transformation of the level of j_{th} attribute possessed by the i_{th} individual expressed as a proportion of the relevant poverty threshold and, consequently, each functioning failure quantified by $\tilde{\pi}$ is a dimensionless number. Thanks to that, the comparability of functioning failures in the different dimensions as well as their combination into a unique number are significantly facilitated.

Two lines along which it is valuable to further narrow $\tilde{\pi}$ down concern the imposition of restrictions on i) the *addenda* of the summation in *i* for a certain dimension *j* – i.e. the poverty levels of different poor individuals regarding dimension *j* – and on ii) the *addenda* of the summation in *j* for a certain poor individual *i* – i.e. her poverty levels in different dimensions. According to the first line, identically to one-dimensional poverty analysis, the behaviour of $p_{i,j}$ as $x_{i,j}$ varies below z_j needs to be chosen. The further $x_{i,j}$ falls below z_j is typically considered as an event yielding either no variation or a proportional increase or a more than proportional increase in *j*-poverty. Individual deprivation functions that are either constant or linearly decreasing or convexly decreasing in $x_{i,j}$ in the interval $\left[0, z_j\right)$ will be used in those three cases, respectively. All decreasing $p_{i,j}$'s accommodate the so-called *Monotonicity* axiom – a decrease in the endowment of attribute *j* induces an increase in *j*-poverty – while convexly decreasing $p_{i,j}$'s satisfy also the so-called *Transfer* axiom since they are sensitive to mean-preserving changes in the distribution of attribute *j* – typically, a 'transfer' between two poor individuals is associated with a poverty increase if the donor is poorer than the recipient.[4] Restricting the functional form of $\tilde{\pi}$ according to

[4] For a deeper analysis and different specifications of the Monotonicity and of the Transfer axioms in a unidimensional setting see Zheng (1997), while for its treatment in a multidimensional setting see Tsui (2002) and Bourguignon and Chakravarty (2003).

the second line entails the incorporation of value judgements concerning the relative importance of different poverty dimensions. Suppose that the endowments of individual *i* in two different dimensions are exactly half the respective poverty thresholds. Should the poverty value in the two dimensions be the same or should one exceed the other? This is the crucial question of 'giving weights' to the different poverty dimensions. More generally, what is implied here is the choice of a rationale for the way the different dimensions should contribute to overall poverty.[5]

Before addressing that issue in the next section let us introduce the *Strong Continuity* axiom (ST), requiring a poverty index to be continuous at (also) the poverty line. Since the poverty value associated with incomes smaller than the poverty line is positive and null otherwise, ST can be accommodated only by indices decreasing towards zero in the left neighbourhood of the poverty line – i.e. $\lim_{t_{i,j} \to 1^-} f(\cdot) = 0$ for all $t_{i,j}$'s. ST has important implications on the way poverty is conceived. A continuous deprivation function is generally justified by the idea that "given a very small change in a poor person's income, we could not expect a huge jump in the poverty level" (Zheng, 1997: 131). However, the belief that this should happen also at the poverty line is disputable. Indeed, "the use of a poverty line to sharply demarcate the rich from the poor suggests […] that a poverty index might be discontinuous at the poverty line" (Donaldson and Weymark, 1986, p. 674). Bourguignon and Fields (1997) conceptualise poverty indices presenting a 'jump' discontinuity at the poverty line as tools able to capture two distinct aspects of the social welfare loss associated with poverty, "the loss from being poor and the loss from being poorer" (p. 155) – for a discussion of the poverty measurement options opened up by this approach, see Esposito and Lambert (2007).

3 Ranking poverty dimensions: restricted vs unrestricted hierarchy

When it comes to the choice of the weighting scheme for the *m* poverty dimensions, two remarks appear necessary. Firstly, as we observed in the introductory section, any scepticism about the choice of prioritising one dimension over another grounded merely on the arbitrary nature of such decision should bear in mind that 'not giving weights' –equivalent indeed to the assignation of identical weights to all dimensions – is itself a subjective decision reflecting the value judgement that those dimensions are equally important. Hence, the informed choice of a weighting system responding to shared value judgements – leading to either equal or unequal weights – should be

[5] 'Giving weights' is not intended here just as a matter of choosing a set of parameters to be used as multiplicative factors for functions $\phi(t_{i,j})$ that are identical across dimensions, but as the choice of functional forms (not necessarily multiplicative) able to quantify poverty in each dimension according to certain value judgements concerning their relative importance.

seen as an inescapable step in a proper multidimensional exercise. Secondly, once two dimensions are deemed to have different importance what usually interests the analyst is to establish a *hierarchy* among them out of any cardinal concern.[6] We envisage two alternative, simple and intuitive ways of conceiving a hierarchy between poverty dimensions *h* and *k*, with dimension *h* deemed to be more important than dimension *k*. Consider the two following requirements:

Unrestricted Hierarchy (UH): $p_h(t_{i,h}) > p_k(t_{i,k})$ *for all* $t_{i,h}, t_{i,k}$.

Restricted Hierarchy (RH): $p_h(t_{i,h}) > p_k(t_{i,k})$, *for all* $t_{i,h}, t_{i,k}$ *such that* $t_{i,h} \leq t_{i,k}$ *but*

$$\exists\, t^*_{i,h} > t^*_{i,k} \text{ such that } p_h(t^*_{i,h}) < p_k(t^*_{i,k})$$

UH requires the poverty value associated with a poor achievement in dimension *h* to be larger than that associated with a poor achievement in dimension *k* independently of the magnitudes of those achievements. In other words, UH requires the poverty value associated with a $z-\varepsilon$ level of achievement in the dominating dimension, $\varepsilon > 0$, to be larger than that associated with a very small (and potentially null) level of achievement in the dominated dimension. Conversely, RH requires the hierarchy to hold only when the poor achievement in dimension *h* is (at least) as large as the poor achievement in dimension *k*.[7] It is highly intuitive to think of requirements UH and RH in terms of percentage of failure in achieving the relevant poverty line. According to UH an *X*% failure in achieving z_h is harsher than a *Y*% failure in achieving z_k for any *X* and *Y*. Differently from UH, RH postulates that an *X*% (or larger) failure in achieving z_h is always harsher than an *X*% failure in achieving z_k, but a large enough failure in achieving z_k is harsher than a small enough failure in achieving z_h. The first kind of ordering will be referred to as $h \overset{UH}{\succ} k$ while the second one as $h \overset{RH}{\succ} k$. The proposed way of sorting out binary comparisons between poverty dimensions is

[6] That is indeed a rather common approach in economics, the most striking case being surely the one of the utility function. Economists have generally been happy with the prescription that an income transfer from individual *v* to individual *u* with $y_v < y_u$ would decrease their combined utilities. The 'right' amount of the decrease is rarely questioned – the most representative attempt to identify the *exact* shape of the utility function being surely Bernoulli (1738, 1954). In a similar fashion, in the development of poverty measures the object of investigation has been more the sign of the variation in the poverty level after changes in poor incomes than its amount.

[7] As remarked in Section 2, to ensure inter-dimension comparability achievements in different dimensions are to be intended as normalised by their respective poverty lines – i.e. as quantified by the ratio $t_{i,j} = \frac{x_{i,j}}{z_j}$.

safely extendible to the organisation of *m*>2 dimensions since it respects the transitivity property of conventional logic – e.g. if $h \overset{UH}{\succ} k$ and $k \overset{RH}{\succ} l$ then $h \overset{UH}{\succ} l$.

In order to investigate the analytical conditions for the accommodation of UH and RH, let us focus our attention on the measure $\tilde{\pi}_i$ yielding the overall poverty value for individual *i*:

$$\tilde{\pi}_i = \sum_{j=1}^{m} p_j\left(t_{i,j}\right). \qquad (2)$$

Firstly, let us note that UH and RH can be interpreted in terms of first order stochastic dominance as the dominant function is required to lay always above the dominated one. Secondly, it can be readily seen that RH is incompatible with headcount-like measures by considering that: i) when both $p_h\left(t_{i,h}\right)$ and $p_k\left(t_{i,k}\right)$ are constant functions and they are not identical, only a $h \overset{UH}{\succ} k$ type of ordering is possible; ii) when only one of $p_h\left(t_{i,h}\right)$ and $p_k\left(t_{i,k}\right)$ is constant we have either $h \overset{UH}{\succ} k$ or an intersection point between the two curves.[8] Hence, in what follows we shall consider only non-constant $p_j(t_{i,j})$'s.

Bourguignon and Chakravarty (2003, p. 37) propose a multidimensional extension of the Foster *et al.* (1984) index – $\pi^{BC} = \frac{1}{n}\sum_{j=1}^{m}\sum_{i \in Q_j} a_j\left(1-t_{i,j}\right)^{\theta_j}$, where $a_j \in \mathbb{R}_{++}$ and the poverty-aversion parameter θ_j is a nonnegative integer number invariant across all *j*'s. This is an example of the implementation of $\overset{RH}{\succ}$ via p_j's exhibiting a multiplicative functional form[9] such as $a_j f\left(t_{i,j}\right)$, where a_j is a weight increasing with the importance assigned to the poverty dimension *j* and for all $\theta_j > 0$ *f* is a poverty-line continuous, strictly positive- and real-valued function decreasing in $t_{i,j}$.

While, as we have just seen, RH and ST can be jointly met by a poverty index, an impossibility theorem can be stated for what concerns the simultaneous accommodation of UH and

[8] Here we do not explore further the case of intersection between the two poverty curves and its possible reading in terms of second order stochastic dominance. In general, the value judgement that at a certain point there is a switch in the hierarchical ordering between dimensions *h* and *k* would require the accommodation of the condition: $\exists\, t'_{i,h} = t'_{i,k}$ such that $p_h(t'_{i,h}) > p_k(t'_{i,k})$ and $\exists t''_{i,h} = t''_{i,k}$ such that $p_h(t''_{i,h}) < p_k(t''_{i,h})$. Given continuity of $p_h(t_{i,h})$ and $p_k(t_{i,k})$ and decreasingness of at least one of them this would imply the existence of an intersection point at $t_{i,h} = t_{i,k} \neq 0$.

[9] Such a functional form is also used by Chakravarty e Silber (2008, p.199).

ST. Consequently, a characterisation theorem for the functional form allowing the accommodation of UH can be directly derived. Those are our Propositions 1 and 2, respectively.

Proposition 1. There exists no poverty index jointly accommodating UH and ST.

Proof. See Appendix A.1.

Proposition 2. The implementation of $h \overset{UH}{\succ} k$ *is allowed if and only if* $p_h\left(t_{i,h}\right)$ *can be written as* $g_h\left(t_{i,h}\right)+\Delta_h$*, where* g_h *is a poverty-line continuous, strictly positive- and real-valued function decreasing in* $t_{i,h}$ *and* $\Delta_h \in \mathbb{R}_{++}$ *is a constant at least as large as* $\max_{t_{i,k}} p_k\left(t_{i,k}\right)$.

Proof. See Appendix A.2.

As a corollary of our Proposition 2, it follows that $h \overset{RH}{\succ} k$ is implemented whenever $p_h\left(t_{i,h}\right)$ first-order dominates $p_k\left(t_{i,k}\right)$ and the magnitude of the jump-discontinuity at the poverty line for $p_h\left(t_{i,h}\right)$ (if any) is smaller than $\max_{t_{i,k}} p_k\left(t_{i,k}\right)$.

Propositions 1 and 2 show that the desired type of hierarchy among poverty dimensions has considerable implications on the way poverty is conceptualised. Proposition 1 shows that the implementation of $h \overset{UH}{\succ} k$ cannot get away from an understanding of poverty which encompasses a 'fixed' welfare loss in dimension *h*, along the interpretation suggested by Bourguignon and Fields (1997) and described in Section 2. More specifically, the $h \overset{UH}{\succ} k$ type of ordering requires not only the existence for dimension *h* of a 'fixed' welfare loss, but also the magnitude of such loss to be at least as large as the poverty value associated with a complete functioning failure in dimension *k* – as required by Proposition 2.

Here is a clarifying example. Take shelter (*sh*) and education (*ed*) as dominating and dominated dimensions, respectively. Suppose that being poor in *sh* means not to have a roof over your head every night, while being poor in *ed* means having less than z_{ed} years of schooling. One may reasonably argue that the fewer nights you manage to wangle a roof over you head the poorer you are, but the 'homeless' condition of not having a guaranteed shelter is *per se* worse than having less than z_{ed} years of schooling – even worse than never having gone to school. Such a value judgement would be reflected in a $sh \overset{UH}{\succ} ed$ type of ordering, which would require $p_{sh}\left(t_{i,sh}\right)$ to exhibit a jump-discontinuity at least as large as $p_{ed}\left(t_{i,ed}=0\right)$.

4 Restricted vs Unrestricted Hierarchy: an empirical application through a simple ordinal approach

4.a) Multidimensional poverty: an ordinal approach to measurement

In order to provide an illustration of the above methodology, among the poverty functions of the form $p_j(t_{i,j}) = g_j(t_{i,j}) + \Delta_j$ let us choose the parametric formulation:

$$\breve{p}_j(t_{i,j}) = \alpha_j \varphi(t_{i,j}) + \omega_j r_j \,, \tag{3}$$

where $0 < \alpha_j < 1$ is a weight increasing with the importance of dimension *j*, φ is a poverty-line continuous, normalised, strictly positive- and real-valued function decreasing in $t_{i,j}$,[10] $\omega_j \in \mathbb{R}_+$ and $r_j \in \mathbb{N}$ denotes dimension *j*'s rank order – poverty dimensions being ranked in increasing order of importance. In (3) the first addendum represents the 'variable loss' from poverty while the second one identifies the 'fixed loss'. The rationale for choosing (3) in our application is that for $\omega_j = \omega \geq 1$ an ordering system of the kind $\overset{UH}{\succ}$ is implemented simply by building upon poverty dimensions' rank order. The reason is that the difference between the rank order of two poverty dimensions that are subsequent in the ordering system is nothing but the difference between two subsequent positive integers, which equals one and hence is able to offset any possible difference between the variable losses of two poverty functions – since $\max_{t_{i,j}} \varphi(t_{i,j}) = \varphi(t_{i,j}=0) = 1$ then $0 < \alpha_j \varphi(t_{i,j}) < 1$ for all *j* and $\left|\alpha_k \varphi(t_{i,k}) - \alpha_h \varphi(t_{i,h})\right| < 1$ for all *h* and *k*. Differently, when ω equals zero for all *j*'s we have first order stochastic dominance among poverty functions which account only for a 'variable loss'. As an outcome, when $\omega_j = 0$ for all *j*'s an ordering of the kind $\overset{RH}{\succ}$ is implemented among poverty dimensions. The overall poverty level of individual *i* is given by:

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[10] Possible φ's are the individual deprivation functions of well-known indices such as those in Foster *et al*. (1984) and Chakravarty (1983).

$$\breve{\mathrm{P}}_i = \sum_{j=1}^{m} \left[\alpha_j \varphi(t_{i,j}) + \omega_j r_j \right] = \left[\alpha_1 \varphi(t_{i,1}) + \omega_1 \right] + \left[\alpha_2 \varphi(t_{i,2}) + 2\omega_2 \right] + ... + \left[\alpha_m \varphi(t_{i,m}) + m\omega_m \right] \quad (4)$$

Note that (4) allows to choose idiosyncratic ω_j's for different poverty dimensions. This opens up the possibility to implement alternative strategies for what concerns the type of hierarchical orderings among dimensions. For example, suppose that the analyst assessing multidimensional child poverty identifies as relevant dimensions 'calories intake' (*ci*), 'housing' (*ho*), 'education' (*ed*) and 'play time' (*pt*), arranged in decreasing order of importance. By setting $\alpha_j = 0.1 r_j$ an ordering of type $\overset{RH}{\succ}$ will be implemented and the functional form in (4) yields:

$$\begin{aligned} \mathrm{P}_i &= \sum_{j=1}^{4} \left[\alpha_j \varphi(t_{i,j}) + \omega_j r_j \right] = \\ &= \left[0.1\varphi_{pt}(t_{i,pt}) + \omega_{pt} \right] + \left[0.2\varphi(t_{i,ed}) + 2\omega_{ed} \right] + \left[0.3\varphi(t_{i,ho}) + 3\omega_{ho} \right] + \left[0.4\varphi(t_{i,ci}) + 4\omega_{ci} \right] \end{aligned}$$

Suppose now that the analyst deems it appropriate to require $ci \overset{RH}{\succ} ho \overset{UH}{\succ} ed \overset{RH}{\succ} pt$. This may be justified by the willingness to implement a $\overset{RH}{\succ}$ type of ordering both among the poverty dimensions related to physical integrity and among poverty dimensions related to psychological development, but a $\overset{UH}{\succ}$ between those two groups. Among the functional forms for individual poverty functions granting the implementation of the hierarchical scheme $ci \overset{RH}{\succ} ho \overset{UH}{\succ} ed \overset{RH}{\succ} pt$, for dimensions *pt*, *ed*, *ho* and *ci* we consider, respectively:

$$\breve{p}_{pt}(t_{i,pt}) = 0.1\varphi(t_{i,pt})$$

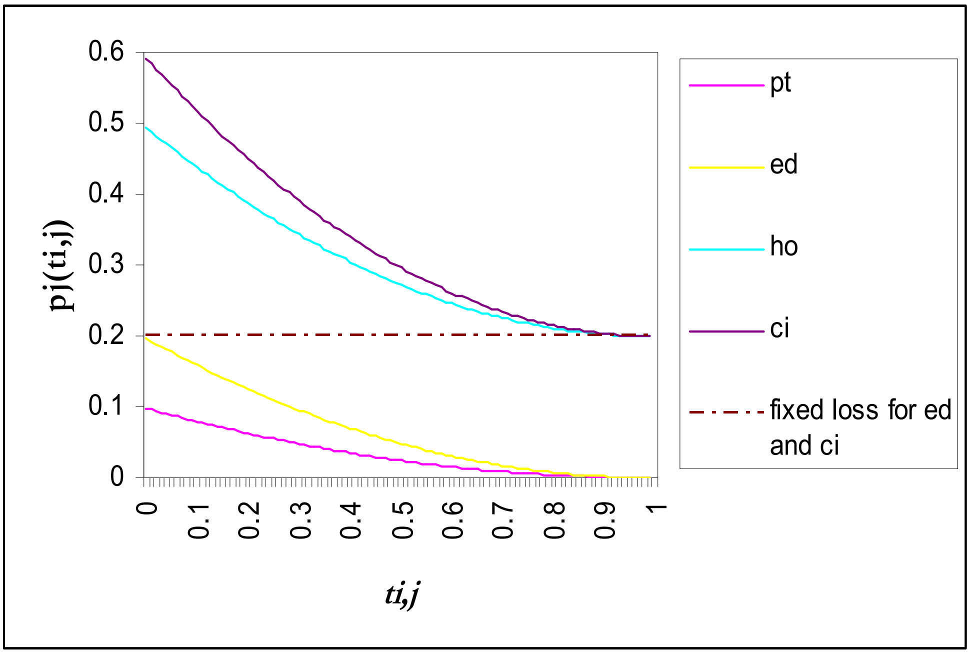
$$\breve{p}_{ed}(t_{i,ed}) = 0.2\varphi(t_{i,ed})$$

$$\breve{p}_{ho}(t_{i,ho}) = 0.3\varphi(t_{i,ho}) + 0.2$$

$$\breve{p}_{ci}(t_{i,ci}) = 0.4\varphi(t_{i,ci}) + 0.2$$

For all dimensions *pt*, *ed*, *ho* and *ci* those functional forms relate to $x_{i,j} < z_j$, while for $x_{i,j} \geq z_j$ the accommodation of the *Focus* axiom clearly requires that $\breve{p}_j(t_{i,j}) = 0$. See Appendix B.1 for the derivation of the conditions on ω_j. In Figure 1 those poverty functions are plotted for $\varphi = \left(1 - t_{i,j}\right)^2$ – squared poverty gap.

Fig. 1: Individual Poverty functions for dimensions *pt*, *ed*, *ho* and *ci*.



4.b) An application to Maldives 1997-2004

The analysis is carried out using data from the Vulnerability and Poverty Assessments, two household surveys run in Maldives Republic in 1997 and 2004 (hereby VPA-1 and VPA-2) by the Minister of Planning and National Development (MPND) with the UNDP collaboration. Basically the same questionnaire and definitions are used in the two waves, making the data fully consistent between the two surveys. All 200 inhabited islands are covered and the sample size is over 2,700 households (2,400 from all atolls, the remaining from Malé)[11].

These surveys provide a wide range of variables regarding living conditions and socio-economic characteristics at both household and individual level, allowing to perform accurate multidimensional poverty and well-being assessment. Moreover, the surveys gather information on the importance attached to the inquired living standard dimensions by both individuals and the Island Communities – the latter being represented by Island Development Committees and Women's Development Committees.[12]

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[11] Both datasets are freely downloadable on the MPND website (www.frdp-maldives.gov.mv/hies/VPA.htm). See also de Kruijk and Rutten (2007).

[12] In particular, in VPA-1 individuals were asked to rank the following 15 domains: 1. Improve the quality of housing; 2. Availability of transport service; 3. Availability of electricity; 4. Communication facilities; 5. Employment opportunity; 6. Possibilities to earn a good income; 7. Food security all year around; 8. Environmental security; 9. Availability of drinking water; 10. Access to consumer goods; 11. Access to health services/improvements; 12. Access

Since our exercise has primarily illustrative purposes, we restrict our poverty analysis to the four top-ranking dimensions according to the aggregate preferences expressed by the population. In decreasing order of importance, those are education (*ed*), health (*he*), housing (*ho*) and employment (*em*). For each of these four dimensions, an indicator of achievement is built upon the information contained in the survey.[13] In our application, an individual is deemed poor in a certain dimension if she is unable to fully function in that dimension – see Appendix C for a more detailed description of the indicators, the corresponding poverty gaps and the relevant aggregation procedure.

We assess poverty in Maldives according to three different specifications of the ordering $ed \succ he \succ ho \succ em$. The first two applications are based on the belief that the ordering among all poverty dimensions is uniquely of a $\overset{RH}{\succ}$ type or of a $\overset{UH}{\succ}$ type, respectively. In the third case, we let the type of ordering depend on the concurrence between the opinion expressed by the population and the one formulated by the Islands' Development Committees. In other words, if people's value judgement that $h \succ k$ is in line with the proclamation of the Committee then we consider that $h \overset{UH}{\succ} k$; if instead the committee does not agree and deems *k* more important than *h* then we opt for $h \overset{RH}{\succ} k$. Since the Committees' ordering is $ed \succ he \succ em \succ ho$, we see that the dominance relations $ed \succ he$ and $he \succ ho$ indeed match those of the population but the ordering of *ho* and *em* is reversed. Hence, in our third empirical application we implement a 'mixed strategy' where $ed \overset{UH}{\succ} he \overset{UH}{\succ} ho \overset{RH}{\succ} em$. Needlessly to say, the criterion we adopt here for the choice of a $\overset{RH}{\succ}$ rather than of a $\overset{UH}{\succ}$ type of ordering is arbitrarily set up to take advantage of the existence of two sources of opinion on the hierarchy among poverty dimensions. Different types of secondary data may suggest different criteria, while the possibility to collect primary data can certainly offer *ad hoc* information; alternatively, the analyst can rely on her own value judgements or on experts' opinions.

In all of our three exercises we set $\alpha_j = 0.1r_j$.[14] For what concerns the choice of ω_j, in our first two applications we set $\omega_j = 0$ and $\omega_j = 1$ for all *j*'s respectively. As to the third application, the poverty functions we adopt for poor achievements in dimensions *em*, *ho*, *he* and *ed* are, respectively (see Appendix B.2 for the derivation of the conditions on ω_j):

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to quality education; 13. Community participation; 14. TV/entertainment facilities; 15. Availability of recreational facilities. VPA-2 adopts the same list but dropping the last three dimensions.

[13] This indicator assumes discrete values ranging from 0 (null achievement) to 4 (full achievement) for all dimensions except for *he*, where levels of achievement equal to 0, 1, 2 are used.

[14] Note that in our exercise $\sum_{j=1}^{m} \alpha_j = 1$ but the analyst can abstract from this restriction.

$$\breve{p}_{em}(t_{i,em}) = 0.1\varphi(t_{i,em})$$

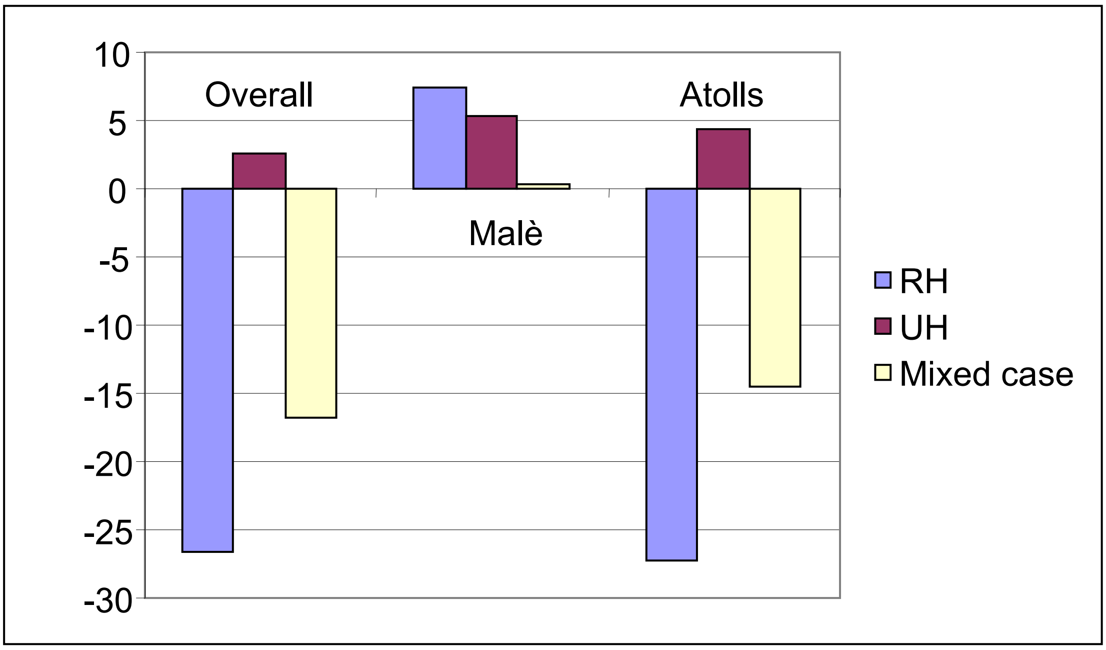
$$\breve{p}_{ho}(t_{i,ho}) = 0.2\varphi(t_{i,ho}) + 0.04$$

$$\breve{p}_{he}(t_{i,he}) = 0.3\varphi(t_{i,he}) + 0.24$$

$$\breve{p}_{ed}(t_{i,ed}) = 0.4\varphi(t_{i,ed}) + 0.54$$

According to our results, the choice of the type of ordering is crucial in the assessment of the multidimensional poverty trend in Maldives. While a poverty *increase* is the response of all of the three applications for what concerns Malè, for the Atolls as well as for the country overall this conclusion is suggested by UH while both RH and the mixed strategy signal a poverty *decrease*. This pattern is robust to the use of the two most commonly used functional forms for the deprivation function $\varphi(\cdot)$ – i.e. the poverty gap (PG) $\varphi = \left(1 - t_{i,j}\right)$ and the squared poverty gap (SPG) $\varphi = \left(1 - t_{i,j}\right)^2$. Further, the results for the country overall hold also for gender subgroups. All figures are presented in Table 1, while Figure 2 provides a graphical illustration of the pattern by area for $\varphi = \text{PG}$.

Fig. 2: Variation in poverty in Maldives (%) by area, 1997-2004, φ=PG



Notes: our elaboration on VPA1-1997 and VPA2-2004 datasets

Table 1. Poverty in Maldives1997 and 2004: φ as PG and SPG for RH, UH and mixed case

	Overall					
RH	PG^{97}=.2364	$>$	PG^{04}=.1867	SPG^{97}=.1447	$>$	SPG^{04}=.0966
UH	PG^{97}=4.4992	$<$	PG^{04}=4.6200	SPG^{97}=4.4075	$<$	SPG^{04}=4.5299
Mixed case	PG^{97}=.6947	$>$	PG^{04}=.5946	SPG^{97}=.6030	$>$	SPG^{04}=.5045
	Decomposition by area					
	Malè					
RH	PG^{97}=.1615	$<$	PG^{04}=.1745	SPG^{97}=.0975	$<$	SPG^{04}=.1142
UH	PG^{97}=3.3740	$<$	PG^{04}=3.565	SPG^{97}=3.310	$<$	SPG^{04}=3.5047
Mixed case	PG^{97}=.4148	$<$	PG^{04}=.4159	SPG^{97}=.3508	$<$	SPG^{04}=.3556
	Atolls					
RH	PG^{97}=.2396	$>$	PG^{04}=.1882	SPG^{97}=.1467	$>$	SPG^{04}=.0943
UH	PG^{97}=4.5478	$<$	PG^{04}=4.756	SPG^{97}=4.4548	$<$	SPG^{04}=4.6621
Mixed case	PG^{97}=.7068	$>$	PG^{04}=.6176	SPG^{97}=.6139	$>$	SPG^{04}=.5237
	Decomposition by gender					
	Female					
RH	PG^{97}=.2356	$>$	PG^{04}=.1841	SPG^{97}=.1454	$>$	SPG^{04}=.0964
UH	PG^{97}=4.423	$<$	PG^{04}=4.5265	SPG^{97}=4.3336	$<$	SPG^{04}=4.4388
Mixed case	PG^{97}=.6833	$>$	PG^{04}=.5812	SPG^{97}=.5931	$>$	SPG^{04}=.4935
	Male					
RH	PG^{97}=.2374	$>$	PG^{04}=.1909	SPG^{97}=.1436	$>$	SPG^{04}=.0969
UH	PG^{97}=4.5955	$<$	PG^{04}=4.7751	SPG^{97}=4.5017	$<$	SPG^{04}=4.6811
Mixed case	PG^{97}=.7093	$>$	PG^{04}=.6168	SPG^{97}=.6155	$>$	SPG^{04}=.5228

Notes: our elaboration on VPA1-1997 and VPA2-2004 datasets

5 Conclusion

Despite the growing interest in multidimensional poverty and wellbeing assessment, thus far very little attention has been paid on how the incorporation of different weighting schemes among dimensions can respond to conflicting value judgements and lead to opposite empirical results. In this paper we present two alternative, simple and highly intuitive ways in which a hierarchy between poverty dimensions can be conceptualised. We show that the implementation of our *unrestricted hierarchy* implies dropping the property of continuity of the poverty index at the poverty line coherently with a conceptualization of poverty as entailing a 'fixed' welfare loss due to poor endowment in a certain dimension.

We derive and fully explain a methodology allowing the implementation of alternative views on a hierarchy criteria which builds upon dimensions' rank order. An application of the proposed methodology to the evaluation of multidimensional poverty for Maldives in 1997 and 2004 shows that the choice of the hierarchical scheme for poverty dimensions can lead to opposite conclusions on the poverty trend.

The reversal of the poverty ordering we obtain is evidently a possible result in all those situations where the condition of first order stochastic dominance between the distributions to be compared is not met. Hence, it is of primary importance to be aware of the potential implications that the choice of the hierarchical scheme among poverty dimensions may have on poverty appraisals by empirical analysts. Further research is needed on both the theoretical and empirical side to increase the options open to the analyst and to test their empirical robustness.

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Appendix A

A..1: Proof of Proposition 1

We shall show that an ordering of the kind $h \overset{UH}{\succ} k$ between dimensions *h* and *k* is incompatible with poverty-line continuous $p_h(t_{i,h})$'s. First, note that $h \overset{UH}{\succ} k$ requires the inequality $p_h\left(t_{i,h}\right) > p_k\left(t_{i,k}\right)$ to be verified for all possible $t_{i,h}$ and $t_{i,k}$. Rearranging the above inequality as $p_h\left(t_{i,h}\right) - p_k\left(t_{i,k}\right) > 0$, we see that zero is the greatest lower bound of the set whose elements are the difference of the generally specified poverty functions for dimensions *h* and *k*, independently of the values of $t_{i,h}$ and $t_{i,k}$. For decreasing poverty functions in the poor domain [0,1) for $t_{i,h}$ and $t_{i,k}$, the condition for $h \overset{UH}{\succ} k$ is equivalent to $\inf_{t_{i,h}} p_h\left(t_{i,h}\right) - \max_{t_{i,k}} p_k\left(t_{i,k}\right) \geq 0$. The proof of Proposition 1 follows easily by contradiction. Suppose that p_h satisfies ST. Then, given continuity, positivity and monotonic decreasingness of p_h it must be that $\lim_{t_{i,h} \to 1} p_h = 0$ equals also $\inf_{t_{i,h}} p_h\left(t_{i,h}\right)$. Now, recalling that p_k is a positive-valued function we are sure that $\max_{t_{i,k}} p_k\left(t_{i,k}\right) = \lambda > 0$. Substituting in the above condition for $h \overset{UH}{\succ} k$, the contradiction inherent in $0 - \lambda \geq 0$ becomes evident. This proves that $h \overset{UH}{\succ} k$ implies poverty-line discontinuous contribution functions for dimension *h*, hence, that a poverty index cannot meet both UH and ST. *Q.E.D.*

A..2: Proof of Proposition 2

The sufficiency side of the proposition is obvious. As to the necessity side, recall that in Proposition 1 the implementation of $h \overset{UH}{\succ} k$ is proved to necessarily imply that $p_h\left(t_{i,h}\right)$ is not continuous at the poverty line. That $p_h\left(t_{i,h}\right)$ must be of the form $g_h\left(t_{i,h}\right) + \Delta_h$ is equivalent to say that the discontinuity of p_h required by Proposition 1 is of the first kind – i.e. a 'jump' discontinuity. Not only this is intuitive, but it also follows necessarily from the considerations that an 'essential' discontinuity is ruled out by the decreasingness and the positivity of p_h and that a 'removable' discontinuity would clearly be inconsequential. Hence, UH can be met only if p_h is an upward translation of some poverty-line continuous function g_h. We now have to prove that for UH to be granted the magnitude of the upward translation $\Delta_{h,k}$ must be at least as large as $\max_{t_{i,k}} p_k\left(t_{i,k}\right)$.

Since $\Delta_{h,k}$ is invariant in $t_{i,h}$ we can substitute $\inf_{t_{i,h}} p_h\left(t_{i,h}\right) = \inf_{t_{i,h}} g_h\left(t_{i,h}\right) + \Delta_h$ in the general condition for the accommodation of UH derived in the proof of Proposition 1 – i.e. $\inf_{t_{i,h}} p_h\left(t_{i,h}\right) - \max_{t_{i,k}} p_k\left(t_{i,k}\right) \geq 0$. Doing so, we obtain $\left[\inf_{t_{i,h}} g_h\left(t_{i,h}\right) + \Delta_h\right] - \max_{t_{i,k}} p_k\left(t_{i,k}\right) \geq 0$, which given continuity of g_h becomes $\left[0 + \Delta_h\right] - \max_{t_{i,k}} p_k\left(t_{i,k}\right) \geq 0$, yielding the desired condition $\Delta_h \geq \max_{t_{i,k}} p_k\left(t_{i,k}\right)$. *Q.E.D.*

Appendix B

Appendix B.1: Derivation of the conditions on ω_j's for the implementation of $ci \overset{RH}{\succ} ho \overset{UH}{\succ} ed \overset{RH}{\succ} pt$:

I. for $ed \overset{RH}{\succ} pt$:

$$\underbrace{\omega_{pt}}_{\text{fixed component for } pt} \leq \underbrace{2\omega_{ed}}_{\text{fixed component for } ed} < \underbrace{\max_{t_{i,pt}} \breve{p}_{pt}\left(t_{i,pt}\right)}_{\max \text{ poverty value for } pt}$$

$$\frac{\omega_{pt}}{2} \leq \omega_{ed} < \frac{0.1\varphi(t_{i,pt} = 0) + \omega_{pt}}{2}$$

II. for $ho \overset{UH}{\succ} ed$:

$$\underbrace{3\omega_{ho}}_{\text{fixed component for } ho} \geq \underbrace{\max_{t_{i,ed}} \breve{p}_{ed}\left(t_{i,ed}\right)}_{\max \text{ poverty value for } ed}$$

$$\omega_{ho} \geq \frac{0.2\varphi(t_{i,ed} = 0) + 2\omega_{ed}}{3}$$

III. for $ci \overset{RH}{\succ} ho$:

$$\underbrace{3\omega_{ho}}_{\text{fixed component for } ho} \leq \underbrace{4\omega_{ci}}_{\text{fixed component for } ci} < \underbrace{\max_{t_{i,ho}} \breve{p}_{ho}\left(t_{i,ho}\right)}_{\max \text{ poverty value for } ho}$$

$$\frac{3\omega_{ho}}{4} \leq \omega_{ci} < \frac{0.3\varphi(t_{i,ho} = 0) + 3\omega_{ho}}{4}$$

There desired ordering does not impose limitations to the admissible values of ω_{pt}. Suppose we choose $\omega_{pt} = 0$ – in order to have a poverty-line continuous deprivation function for dimension *pt*. The condition on ω_{ed} for the implementation of $ed \overset{RH}{\succ} pt$ becomes $0 \leq \omega_{ed} < \frac{0.1\varphi(t_{i,pt} = 0)}{2}$ $\Rightarrow$

$0 \le \omega_{ed} < \frac{0.1}{2}$. Any value of ω_{ed} in the specified interval is indifferent as to the implementation of $ed \overset{RH}{\succ} pt$. Similarly to $\breve{p}_{pt}$, if we want $\breve{p}_{ed}$ to exhibit only a 'variable loss' we would go for $\omega_{ed} = 0$. The condition for $ho \overset{UH}{\succ} ed$ would then become $\omega_{ho} \ge \frac{0.2}{3}$. Finally, if we opt for $\omega_{ho} = \frac{0.2}{3}$ then $ci \overset{RH}{\succ} ho$ will require $\frac{0.2}{4} \le \omega_{ci} < \frac{0.5}{4}$. Once also the value for ω_{ci} is selected, say $\omega_{ci} = \frac{0.2}{4}$, the individual poverty functions for the four dimensions are fully determined.

Appendix B.2: Derivation of the conditions on ω_j's for the implementation of $ed \overset{UH}{\succ} he \overset{UH}{\succ} ho \overset{RH}{\succ} em$:

1. for $ho \overset{RH}{\succ} em$: $$\underbrace{\omega_{em}}_{\text{fixed component for em}} \le \underbrace{2\omega_{ho}}_{\text{fixed component for ho}} < \underbrace{\max_{t_{i,em}} \breve{p}_{em}\left(t_{i,em}\right)}_{\max \text{ poverty value for em}}$$
$$\frac{\omega_{em}}{2} \le \omega_{ho} < \frac{0.1\varphi(t_{i,em} = 0) + \omega_{em}}{2}$$

2. for $he \overset{UH}{\succ} ho$: $$\underbrace{3\omega_{he}}_{\text{fixed component for he}} \ge \underbrace{\max_{t_{i,ho}} \breve{p}_{ho}\left(t_{i,ho}\right)}_{\max \text{ poverty value for ho}}$$
$$\omega_{he} \ge \frac{0.2\varphi(t_{i,ho} = 0) + 2\omega_{ho}}{3}$$

3. for $ed \overset{UH}{\succ} he$: $$\underbrace{4\omega_{ed}}_{\text{fixed component for ed}} \ge \underbrace{\max_{t_{i,he}} \breve{p}_{he}\left(t_{i,he}\right)}_{\max \text{ poverty value for he}}$$
$$\omega_{ed} \ge \frac{0.3\varphi(t_{i,he} = 0) + 3\omega_{he}}{4}$$

Among the possible ω_j's respecting the above conditions we choose $\omega_{em} = 0$, $\omega_{ho} = 0.02$ (because, for some reasons, we want a poverty-line continuous deprivation function for *em* but a poverty-line discontinuous deprivation function for *ho*), $\omega_{he} = 0.08$ and $\omega_{ed} = 0.135$.

Appendix C

Variables description

The four indicators have been constructed according to the following lines: a) *housing* is the outcome of three main indicators: available living space per capita, availability of rain water storage facility, and days of lack of drinking water; b) *health* is described in terms of possibility to get medicine when necessary; c) *education* is measured in terms of educational attainment and d) *employment* is the measured with reference to the condition of unemployment or under-employment. While housing and health are determined at household level, education and employment refer to the individual. More details can be found in the table below:

Elementary variables, composite indicators and corresponding poverty gaps.

Indicators	Variables	Variable description	Poverty gap	Aggregation
HOUSING	Living space per capita	Sq.feet occupied by the household divided by number of household component (SQF)	≤ 40 SQF = 1 41-60 SQF =.75 61-100 SQF =.50 101-200 SQF =.25 > 200 SQF[1] =0	Intersection & Union: Povhousing=1 if both SQF and DLW=1; Povhousing=.75 if SQF or DLW=.75 Etc.
	Drinking water	No water storage because cannot afford (NWS) Days of insufficient drinking water experienced (DLW)	NWS & 90 LWD = 1 > 90 LWD =.75 30-90 LWD =.50 29- 10 LWD =.25 < 10 LWD =0	
HEALTH	Access to medicine	Getting medicine (GM) or not getting medicine (NGM) when necessary Reasons for not: cannot afford (CA), not available on the island (NA)	NGM & CA =1 NGM&NA =.50 Otherwise =0	
EDUCATION	Education level	Highest level of education achieved	None/illiterate =1 Read/write only =.75 Functional literacy =.50 Local certificate =.25 Otherwise =0	
EMPLOYMENT	Unemployment and underemployment	Unemployment (U) No earners in the family (NE) At least one earner (OE) N° of months/year of work (EM)	U & NE =1 U & OE =.75 EM ≤ 1 =.50 EM 2-11 =.25 Otherwise =0	

[1] 200 sq. feet corresponds to the median value